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Superfluidity and Superconductivity in the Universe

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The paper aims to elucidate the current status of the problem concerning the existence and observation of superfluid and superconducting states in the universe, that is, under cosmic conditions. Following an introduction, the paper discusses Bose–Einstein condensation, superfluidity, and superconductivity; possibilities for the occurrence of superfluidity and superconductivity under cosmic conditions; superconductivity of dense, degenerate electron plasma (large planets, white dwarfs); superfluidity and superconductivity in neutron stars; and finally superfluidity in a cosmological neutrino "sea."

KEY WORDS: Superfluidity; Superconductivity; Neutron star; White dwarf; Cosmology.

1. INTRODUCTION

The phenomena of superfluidity and superconductivity were discovered in the laboratory and have not yet been observed on the earth under natural conditions. This circumstance is quite understandable because for all known substances superfluidity and superconductivity can occur only at rather low temperatures.

Liquid helium (the He^4 isotope) becomes superfluid for $T \leqslant T_\lambda$, where $T_\lambda = 2.17°K$ is the temperature of the λ-transition from He I to He II. In addition to He^4, superfluidity is observed for solutions of He^3 in He^4, but again only at a low temperature (when He^3 is added to He^4 the temperature of the λ-transition drops). Pure He^3 would presumably become superfluid only at a temperature considerably

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closer still to absolute zero (at $T < 0.01°K$). At comparatively high temperatures, so far as is known, the only hope for observing superfluidity under terrestrial conditions would be in the case of an "exciton fluid" in semiconductors.[1] But such a "fluid" would be extremely short-lived, since its electron and hole components would annihilate, for example, with the radiation of light. Thus one could hardly expect to observe it without specially designed equipment.

Superconductivity has been detected in a very large number of metals and alloys, but in all known cases the critical temperature T_c (for $T > T_c$ the material is no longer superconducting) does not exceed 21°K. In principle, the production of high-temperature superconductors would be possible, or at any rate cannot be excluded.[2] This, however, is quite another question. Here we merely wish to emphasize that superfluidity and superconductivity have thus far been detected only in the range of low temperatures, which must be established artificially.

A natural question arises: might superfluidity and superconductivity exist away from the earth? There are certainly regions in the universe where the temperature is very low; but the temperature will ordinarily be higher than some tens of degrees, or at least several degrees, Kelvin. Most important, there is no reason to expect droplets of liquid helium to exist in space, nor in most situations any superconducting bodies or particles. In view of this circumstance as well as entirely independently, the problem of detecting superfluidity and superconductivity in the universe takes a completely different form: might superfluid or superconducting states of new, or properly, of "nonterrestrial" types and for "nonterrestrial" substances exist under cosmic conditions?

This question is in fact being discussed for several years, and it gradually became clear that the problem of superfluidity and superconductivity in the universe is not merely a theoretical curiosity but is actually of genuine astrophysical interest. We shall attempt below to elucidate the current status of the problem.

2. BOSE–EINSTEIN CONDENSATION, SUPERFLUIDITY, AND SUPERCONDUCTIVITY

"In the beginning there was an ideal Bose gas," or "in the beginning there was a Bose–Einstein condensation"—such epigraphs would be perfectly appropriate in books dealing with the theory of superfluidity and superconductivity. In fact, although at one time there was some doubt, it has now become sufficiently clear that Bose–Einstein condensation does represent the basis of the superfluidity and superconductivity phenomena (at any rate this claim applies to known and investigated cases).

We may recall that the total number N of particles in an ideal Bose gas occupying the volume V is given (see, for example, Reference 3) by

$$N = \frac{gVm^{3/2}}{2^{1/2}\pi^2\hbar^3} \int_0^\infty \frac{\sqrt{E}\, dE}{e^{(E-\mu)/kT} - 1} \tag{1}$$

Here $E = p^2/2m$ is the energy of a nonrelativistic particle, g is the statistical weight ($g = 2S + 1$, where S is the spin), and μ is the chemical potential.

If the condition

$$\frac{N}{V}\left(\frac{\hbar}{mkT}\right)^{3/2} \ll 1 \tag{2}$$

is satisfied, the behavior of the Bose gas will not differ from the classical case; that is, Boltzmann statistics will be valid. But if the inequality (2) is violated, the pressure of the Bose gas will become lower than the pressure of a classical gas, corresponding to a certain attraction between the particles resulting from the quantum-mechanical exchange effect. If the temperature drops still further, so that the violation of the condition (2) becomes even stronger, Bose–Einstein condensation will set in.

For a given particle concentration $n = N/V$, Eq. (1) will determine the chemical potential μ, and as is readily seen the potential μ for a Bose gas will always be negative or zero, because of the positive value of the quantity N/V. We will have $\mu = 0$ if [Eq. (1), with the substitution $E/kT = z$]

$$n \equiv \frac{N}{V} = \frac{g(mkT_0)^{3/2}}{2^{1/2}\pi^2\hbar^3}\int_0^\infty \frac{\sqrt{z}\,dz}{e^z - 1} \tag{3}$$

From this equation we can determine the temperature of the Bose–Einstein condensation:

$$T_0 = \frac{3.31}{g^{2/3}}\frac{\hbar^2}{mk}\left(\frac{N}{V}\right)^{2/3} \tag{4}$$

If $T < T_0$, Eq. (1) has no negative solutions for μ; this is due to the inapplicability of Eq. (1) under conditions where particles accumulate on the lower level with energy $E = 0$. But as $T \to 0$ all the particles in an ideal Bose gas should be concentrated on just this lower level. For temperatures $T > T_0$ the lower level plays no significant role, but for $T \leqslant T_0$ the macroscopically large number of particles

$$N(T) \equiv N(E = 0) = N[1 - (T/T_0)^{3/2}] \tag{5}$$

will be found on the lower level $E = 0$. At the same time the remaining particles will be located on all the other levels having $E > 0$, with the populations

$$N(E > 0) = N - N(E = 0) = N(T/T_0)^{3/2}$$
$$= \frac{gV(mkT)^{3/2}}{2^{1/2}\pi^2\hbar^3}\int_0^\infty \frac{\sqrt{z}\,dz}{e^z - 1} \tag{6}$$

Thus for $T \leqslant T_0$ Bose–Einstein condensation will take place—the accumulation of a finite (not an infinitesimal) portion of the particles in an ideal Bose gas on the lower level with energy $E = 0$ and momentum $p = 0$. In other words, one may say that in an ideal Bose gas with $T \leqslant T_0$ the particle distribution function with respect to momentum will have the form

$$N(\mathbf{p}) = N(T)\,\delta(\mathbf{p}) + f(\mathbf{p})$$

where $N(T)$ is given by Eq. (5), δ represents the delta-function, and $f(\mathbf{p})$ is some smooth function of $\mathbf{p}$ or, more accurately, of the quantity $p = \sqrt{2mE}$.

How will interaction between the particles, that is, the transition to a nonideal Bose gas or fluid, influence the Bose–Einstein condensation? Logically it would be possible for the level with $\mathbf{p} = 0$ to cease to be a distinguished level in the presence of interaction, even for $T = 0$. Generally speaking, however, one finds that this is not the case: the influence of interaction, at any rate so long as it remains weak, will lead to a decrease in the number of particles having zero momentum, but as before for $T < T_0$ (where T_0 represents some temperature of the condensation or the transition) the particle distribution function will have the form

$$N(\mathbf{p}) = N(T)\,\delta(\mathbf{p}) + f(\mathbf{p}) \tag{7}$$

However, if in an ideal gas $N(T = 0) = N$, then in the presence of interaction we will have $0 < N(T = 0) < N$, where N is the total number of particles. For a sufficiently strong interaction, corresponding to attraction at large distances, the gas cannot be cooled to the condensation temperature T_0, for it will previously have been converted into a liquid or a solid body. A single-component solid body such as solid helium or, say, solid neon, cannot of course be superfluid. Only helium remains liquid as $T \to 0$ if the pressure is not too high. Liquid helium, if we are speaking of the He^4 isotope, will obey Bose statistics and will be superfluid throughout the temperature range $T < T_0 \equiv T_\lambda = 2.17°K$. Helium will remain liquid even at absolute zero[2] as a result of the large zero-point vibrations of light He atoms and since the interaction between atoms is at the same time comparatively weak. Both these factors apply equally to He^4 and to He^3. But liquid He^3 is not superfluid, at least at temperatures above 0.01°K. From this fact alone it is natural to conclude that the superfluidity of liquid He^4 is associated with the Bose statistics of He^4 atoms, while He^3 atoms obey Fermi statistics. Moreover, a theoretical analysis indicates (we are, to be sure, speaking merely of estimates) that in luiquid He^4 as $T \to 0$ about 10% of all the particles will have a vanishing momentum. This result has not yet been established reliably by direct experiment, for example, on neutron scattering, but all the available theoretical and experimental data on liquid helium support this conclusion. Thus in a superfluid liquid, as in a Bose gas, $N(T) \neq 0$ in a relation such as Eq. (7). At the λ-point, where $T = T_\lambda$, we have $N(T_\lambda) = 0$, and in a nonsuperfluid He II liquid, which exists in some temperature range $T > T_\lambda$, $N(T) = 0$.

There is no space here to dwell in further detail on the relation between Bose–Einstein condensation and the superfluidity phenomenon. One usually considers that superfluidity is possible only for $N(T) \neq 0$, that is, if a macroscopic number of particles are present in the state with momentum $p = 0$ (Bose–Einstein condensation or condensation in momentum space). Nevertheless, the author should point out that he is not aware of an entirely rigorous proof of this assertion, nor of the conclusion that in any conceivable liquid of Bose particles we always have $N(0) \neq 0$, that is,

[2] Here we are of course extrapolating the data referring to the region of very low but nonzero temperatures. No factors are as yet known that would prevent us from making such an extrapolation.

that Bose–Einstein condensation will occur. Furthermore, models can evidently be developed in which the situation is more complicated.(4) For the cases of interest to us, however, there is clearly no reason to go beyond the scope of the ideas mentioned above, and we shall henceforth make the assumption, without special stipulation, that matter is superfluid if it remains liquid and if Bose–Einstein condensation takes place within it.[3]

The superconductivity phenomenon was long ago characterized as the superfluidity of an electron fluid in metals. However, the character of the relationship between the two effects "at the molecular level" remained unclear. It was moreover found that electrons obeying Fermi statistics can in no way participate in Bose–Einstein condensation. When the microscopic theory of superconductivity was developed in 1957 it nevertheless became evident that superconductivity is essentially a close ally of Bose–Einstein condensation in a Bose gas. Indeed, if mutual attraction is present two electrons located near a Fermi surface and having opposite momenta and opposite spins will "adhere" in a pair with charge $2e$ and spin zero (the latter circumstance is in principle unimportant, since a "coupling" of electrons with parallel spins would also have led to the formation of a complex particle, a pair with integral spin). Such pairs, which are analogous to the atoms of positronium or to electron–hole pairs in semiconductors,[4] will, under conditions where they may be regarded as an individual particle (like an α-particle), obey Bose statistics and should undergo Bose–Einstein condensation. Thus superconductivity is associated with the Bose–Einstein condensation of electron pairs. The superfluidity of such a system will be manifested as superconductivity because the pairs are charged, so that their flow will entail not only a transfer of mass but a transfer of charge, that is, an electric current.

As applied to actual metals, the concept of Bose–Einstein condensation of electron pairs is a rather conventional one, as the diameter of the pairs is considerably greater than the distance between them. But between the Bose–Einstein condensation of an ideal Bose gas and the transition to the superfluid state of liquid He^4 the distance mentioned does not have a small value. In any event both the superfluidity of He II and the superconductivity of metals are based on the same phenomenon—Bose–Einstein condensation.

The formation of electron–electron pairs in a metal may at first glance seem a very strange and even an incomprehensible phenomenon. With the development of the microtheory of superconductivity (the theory of Bardeen, Cooper, and Schrieffer, or briefly the BCS theory; see References 5, 6), however, it was found that pair formation near a Fermi surface is by no means exotic. In the first place, two particles moving near a Fermi surface in a degenerate gas will form a bound state (will "adhere") however weak the attraction between them. This result is analogous to the

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[3] An ideal Bose gas would not be superfluid even though it may undergo Bose–Einstein condensation, because its flow would be unstable relative to the formation of "excitations" (in other words, the well-known Landau superfluidity criterion would not be satisfied in this case; see Sec. 67 of Reference 3). But in an even slightly nonideal Bose gas, as Bogolyubov once showed, the Landau criterion will be satisfied for sufficiently small velocities of motion, and superfluidity will be possible.

[4] The main distinction is that a pair of two electrons is charged, whereas a positronium atom and an electron–hole pair are neutral.

familiar fact that a bound level will exist in a two-dimensional potential well, no matter how shallow (for further details regarding this analogy, see the author's recent paper[2]). Secondly, electrons in a metal can attract each other if the distance between them falls within a certain range, because for a certain range of values of the frequency ω and the wave vector $\mathbf{q}$ the dielectric constant $\epsilon(\omega, \mathbf{q})$ in a metal is negative. To be specific, the negative contribution to ϵ may arise from the influence of the crystal lattice, or in quantum language, it may be associated with phonon exchange. It is worth noting that, as has recently been pointed out, [7] a "coupling" of electrons is also possible for repulsive forces provided they are equal to zero at the Fermi boundary itself. It is then necessary to consider second-order processes leading to an effective attraction between electrons independently of the sign of the interaction energy. This is just the situation in the well-known "jelly" model,[6,7] for which

$$\epsilon(\omega, q) = 1 - \frac{\omega_{0i}^2}{\omega^2} + \frac{\kappa^2}{q^2}$$

Here

$$\omega_{0i} = \sqrt{\frac{4\pi e^2 Z^2 n_i}{M}} = \sqrt{\frac{4\pi e^2 Z n}{M}}$$

is the plasma frequency for ions of mass M, charge eZ, and concentration $n_i = n/Z$, with n the electron concentration. Furthermore $\kappa^2 = 6\pi n e^2/E_F$; E_F is the Fermi energy and $1/\kappa$ is the screening radius. The interaction between the electrons, if it is a first-order effect, is described by the Fourier components

$$V(\omega, \mathbf{q}) = \frac{4\pi e^2}{q^2 \epsilon(\omega, q)}$$

where $\hbar\omega$ and $\hbar\mathbf{q}$ are the variations in the energy and momentum of an electron arising through phonon exchange.

In the original version of the BCS theory the simplest approximation was used, the electrons being regarded as attracted near the Fermi surface within a layer of thickness $\hbar\omega_c \ll E_F$, with an effective interaction independent of ω and $\mathbf{q}$ (for further details, see References 5, 6):

$$\begin{aligned} V(\omega, q) &= -V > 0, & \omega &\leqslant \omega_c \\ V(\omega, q) &= 0, & \omega &> \omega_c \end{aligned} \tag{8}$$

The gap $\Delta(T)$ in the electron spectrum depends on the temperature and "closes" (disappears) at the critical temperature

$$T_c = \frac{\Delta(0)}{1.76k} = \theta e^{-1/g}, \qquad g = N(0) \cdot V \tag{9}$$

Here $\Delta(0)$ is the width of the gap for $T = 0$ (we note that an energy 2Δ must be lost in order for a pair to break up, since the energy Δ refers to a single electron), the temperature $\theta \approx \hbar\omega_c/k$, and for the phonon attraction mechanism $\theta \approx \theta_D$, where θ_D is the Debye temperature for the given metal. Moreover V is the effective interaction (8) and $N_0 \equiv N(E_F)$ is the density of the levels of electrons having a particular projection of their spin near the Fermi surface, that is, with energy E_F. We are speaking here of the normal state of the system; for free electrons

$$E_F = \frac{\hbar^2 k_F{}^2}{2m}, \qquad n = \frac{k_F{}^3}{3\pi^2}, \qquad N(0) = \frac{1}{2}\left(\frac{dn}{dE}\right)_{E_F} = \frac{k_F}{2\pi^2\hbar^2}$$

In the BCS theory the constant g is not calculated. [Equation (9) is of course meaningful only for $g > 0$; if $g < 0$ the metal will not be superconducting even at obsolute zero.] It seems highly likely that we will always have $g \lesssim 1$, and for simple models $g \leqslant \frac{1}{2}$ (see References 2 and 6, and the references cited there). However, for values $g \sim \frac{1}{2}$, corresponding to strong binding, Eq. (9) is no longer accurate; it refers to the region of weak binding, where $g \ll 1$. In most cases the binding may be considered weak, and in any event this question will not be of importance in the sequel. We may therefore adopt the BCS equation (9), particularly since the parameters θ and g should be refined in any particular instance.

The author has hoped that the present paper might be of interest to physicists and astrophysicists whose work is remote from the problems of low-temperature physics. For this reason it was thought opportune briefly to discuss above the relation between Bose–Einstein condensation, superfluidity, and superconductivity. From the stand-point of an analysis of the conditions for the superfluidity and superconductivity phenomena in space, it is particularly vital to stress that these phenomena can occur both for a collection of bosons and for a degenerate system of fermions under circumstances where interparticle attraction exists.

3. POSSIBILITIES FOR THE OCCURRENCE OF SUPERFLUIDITY AND SUPERCONDUCTIVITY UNDER COSMIC CONDITIONS

Apart from unstable particles, bosons may include certain nuclei, atoms and molecules, and also photons and gravitons (quanta of the gravitational field).

With regard to the atomic and molecular bosons, it is recognized that they will form a solid body before Bose–Einstein condensation has set in. Liquid helium is an exception. In other words, we have nothing new here in comparison with the conditions in the terrestrial laboratory. Moreover, liquid helium in all likelihood is not to be found anywhere in the cosmos.

For the atomic-nucleus bosons, however, the "cosmic possibilities" are incomparably wider than on the earth. At sufficiently high temperatures the nuclei will be freed from their electron shells and will form a heavy component of cosmic plasma, as for example in stellar interiors. The most interesting particles from this standpoint are helium nuclei, or α-particles (we are referring to He^4). For a gas of α-particles the characteristic temperature T_0 for condensation is given [see Eq. (4)] by

$$T_0 = 3.31 \frac{\hbar^2}{M_{\mathrm{He}}^{5/3} k} \rho^{2/3} \approx 11\rho^{2/3}\ {}^\circ\mathrm{K} \tag{10}$$

where $M_{\mathrm{He}} = 6.7 \times 10^{-24}$ g is the mass of an α-particle and $\rho = M_{\mathrm{He}}N/V$ is the gas density in grams per cubic centimeter.

If the α-particles in a stellar interior form a nonideal gas or liquid, then for $T < T_\lambda \sim T_0$ this system will be superfluid, or more accurately superconducting, because the transfer of α-particles will induce a current (the author first learned of this possibility from M. Ruderman). For the values of ρ and $T < T_\lambda$ considered here, however, the helium atoms must be virtually fully ionized, while on the other hand they must not form a crystal lattice.[5]

For a cool substance ionization will take place upon strong compression, and may be considered complete[(3)] if

$$\rho \gg \left(\frac{me^2}{\hbar^2}\right)^3 \mu_e M_p Z^2 \approx 20Z^2 \text{ g/cm}^3 \tag{11}$$

Here Z is the atomic number of the nucleus, m is the mass of an electron, and $\mu_e M_p$ is the mass of the material referred to a single electron, since $\rho = n\mu_e M_p$ (n is the electron concentration and M_p is the mass of a proton). For helium we have $\rho \gg 80$ g/cm^3, and by Eq. (10), $T_0 \gg 200$°K.

On the other hand, material that is compressed sufficiently strongly will crystallize[(8,9)] and the melting temperature T_m will be determined by the condition $kT_m = \Gamma_m^{-1}(eZ)^2/\bar{r}_i$, where $(4\pi\bar{r}_i^3/3)^{-1} = n_i$ is the ion concentration and Γ_m is a numerical parameter. Thus melting will set in when the kinetic energy kT per degree of freedom of the oscillator is Γ_m times smaller than the Coulomb interaction energy $eZ^2/\bar{r}_i$ of the nuclei. According to recent calculations[(10)] for a simple plasma model, $\Gamma_m = 170 \pm 10$ and

$$T_m \approx 10^3 Z^{5/3} \rho^{1/3} \text{ °K} \tag{12}$$

where we have set $\rho = 2ZM_p n_i$ and $M_p = 1.67 \times 10^{-24}$ g is the mass of a proton; for $Z = 2$ we evidently have $T_m \approx 3 \times 10^3 \rho^{1/3}$. Comparing the values of T_0 and T_m for helium, we see that $T_0 \leqslant T_m$ for $\rho \leqslant 3 \times 10^7$ g/cm^3. Thus superfluidity phenomena can be expected only for densities $\rho \geqslant 3 \times 10^7$ g/cm^3 [in which event, according to Eq. (10), $T_0 \geqslant 10^6$]. Such densities would be possible only in the central parts of white dwarfs having a mass close to the limiting mass $M_{\mathrm{cr}} \approx 1.2 M_\odot$ (for masses $M > M_{\mathrm{cr}}$ a cool star will either be transformed into a neutron star or will collapse; see, for example, Reference 11). It is evidently possible, then, for superconducting nuclei (specifically, He^4 nuclei) to appear in the central regions of stars. The analysis that has been made, however, cannot be considered entirely convincing, and the problem is in need of further study. Nevertheless it would seem that the range of values of the parameters ρ and T for which superfluidity (superconductivity) is

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[5] We further assume that no type of collective bound state of the α-particles with the electrons is formed (see Reference 1).

possible would not in any event be appreciably extended. If this is indeed the case then He^4 nuclear superfluidity, to say nothing of that of other nuclei, could occur only in very rare cases.

Of the remaining bosons we have finally to examine the possibility of Bose–Einstein condensation of photons and gravitons. At first sight the mere formulation of such a question might seem to involve a misunderstanding. In fact, photons and gravitons are always particles relativistic in the limit (rest mass equal to zero), and accordingly their number would not be conserved even in the low-temperature region. Hence in a state of thermodynamic equilibrium the chemical potential μ of a photon gas (that is, blackbody radiation) or a graviton gas would be equal to zero. Properly, it is from the condition $\mu = (dF/dN)_{T,V} = 0$ that the Planck distribution is obtained. We therefore may not speak of an equilibrium Bose–Einstein condensation of photons or gravitions. It is perfectly legitimate, however, to inquire about Bose–Einstein condensation under conditions of incomplete equilibrium. For example, the excitons in a solid body live for only a finite time and then disappear because of transformation into radiation or an aggregate of phonons (for definiteness we have in mind excitons of the type of a bound electron–hole system). But if the time required to establish equilibrium and, in particular, condensation in a system of excitons is substantially shorter than the lifetime of the excitons, it will be entirely meaningful to speak of condensation and superfluidity in the exciton gas.[(1)] In just the same way, if the scattering of photons plays a significantly greater part than the process of photon production and absorption, one could consider the Bose–Einstein condensation of photons. Free electrons can only scatter light; they cannot absorb or emit it. Thus scattering will make a particularly strong contribution in a rarefied electron plasma. Nevertheless, the absorption and emission of light that takes place when electrons collide with ions is also appreciable, and in general one cannot speak of any well-defined Bose–Einstein condensation. The interaction of gravitons with matter is very weak, and the problem of the establishment of equilibrium in a system of gravitons, as well as the influence of Bose statistics on this process, would probably be of interest only in models of an expanding universe when states of extremely high density are considered.

In the case of bosons, then, superfluidity or superconductivity under cosmic conditions is practically impossible, except for the superconductivity of nuclei in the interiors of white dwarfs. Accordingly, in connection with the topic of this paper we will be interested almost exclusively in superconductivity or superfluidity in Fermi systems, analogous to electron superconductivity in metals. The following actual possibilities obtain:

1. Superconductivity could occur in the dense, degenerate, metallic-type electron plasma that exists in large planets such as Jupiter and Saturn, in white dwarf stars, and at the periphery of neutron stars.

2. The neutron fluid representing the principal component of the material in neutron stars should be superfluid within a certain fairly wide density range. The proton fluid admixed with the neutron fluid might prove to be superconducting.

3. One cannot entirely exclude the possibility of superfluidity in a neutrino "sea"—a degenerate gas of neutrinos that can exist in early evolutionary stages in certain cosmic models.

We shall proceed to discuss all three of these cases.

4. SUPERCONDUCTIVITY OF DENSE, DEGENERATE ELECTRON PLASMA (LARGE PLANETS, WHITE DWARFS)

Under sufficiently strong compression, even at a temperature equal to zero (that is, as $T \to 0$), matter will be transformed into a metallic (conducting) state through collectivization of orbital (bound) electrons. In particular, if the condition (11) holds the material will certainly be metallized. It is very important here to note that as the density increases an electron gas will approach increasingly close to an ideal gas, so that a model of weakly bound electrons will be found more applicable than in ordinary metals. In fact, the zero-point (kinetic) energy of the electrons in a degenerate ideal Fermi gas will be

$$K \approx \frac{\hbar^2}{2m\bar{r}^2} \approx \frac{\hbar^2}{m} n^{2/3}$$

where $n = N/V$ is the electron concentration; evidently in order of magnitude we have

$$K \approx E_F = \frac{\hbar^2 k_F{}^2}{2m} = \frac{mv_F{}^2}{2} = (3\pi^2)^{2/3} \frac{\hbar^2}{2m} n^{2/3}$$

At the same time the energy of Coulomb interaction between electrons will be $U_{ee} \approx e^2/\bar{r} \approx e^2 n^{1/3}$, while the energy of interaction between an electron and a nucleus (of charge eZ) will be $U_{ei} \approx Ze^2/\bar{r}_{ei} \approx Z^{2/3}e^2n^{1/3}$, where $\bar{r}_{ei} \approx n_i^{-1/3} = (Z/n)^{1/3}$. Evidently

$$\frac{U_{ei}}{K} \approx Z^{2/3} \frac{U_{ee}}{K} \approx Z^{2/3} \frac{e^2 m}{\hbar^2} n^{-1/3} \approx Z^{2/3} \frac{e^2}{\hbar v_F} \tag{13}$$

where the velocity at the Fermi surface is

$$v_F = \sqrt{2E_F/m} = (3\pi^2)^{1/3} \hbar n^{1/3}/m$$

It is clear from Eqs. (13) that the ratio of the interaction energy and kinetic energy declines with increasing n according to an $n^{-1/3}$ law, that is, a $\rho^{-1/3}$ law.

Unfortunately, even for strongly compressed material, when $U_{ei}/K \ll 1$, and *a fortiori* $U_{ee}/K \approx e^2/\hbar v_F \ll 1$, the problem of superconductivity has not yet been definitely resolved. As a matter of fact in a degenerate Fermi gas it is difficult to determine the sign of the interaction forces between electrons near the Fermi surface. This is also the case for strongly compressed material, since even under such conditions a major contribution arises from the interaction of electrons with phonons having large momenta (that is, with small wavelengths, comparable to the distance between nuclei; a quantitative theory has not yet been developed for short-wave phonons of this kind). If, however, it is attraction that takes place, then to estimate T_c it would in general be appropriate to use the BCS equation (9) and the formulas obtained for the parameter $g = N(0) \cdot V$ for simple models. The "jelly" model mentioned previously is more suitable for strongly compressed material than in other cases,[7] for under strong compression the role of transverse phonons and "umklapp" processes will be suppressed. For the "jelly" model[7] we have

$$T_c \approx \frac{e^2}{kv_F}\,\omega_{0i}\exp\left(-\frac{8\hbar v_F}{\pi e^2}\right) \approx 3\times 10^3\rho^{1/6}\exp(-2.8\rho^{1/3})\ {}^\circ\mathrm{K} \tag{14}$$

$$\begin{aligned} \omega_{0i} &= \sqrt{\frac{4\pi e^2 Zn}{M}} = \sqrt{\frac{2\pi e^2 n}{M_p}} = 9\times 10^2\sqrt{n} = 4.9\times 10^{14}\rho^{1/2}\ \mathrm{sec}^{-1} \\ v_F &= (3\pi^2)^{1/3}\frac{\hbar}{m}n^{1/3} = 3.6n^{1/3} = 2.4\times 10^8\rho^{1/3}\ \mathrm{cm/sec} \\ \rho &= \frac{n}{Z}M = 3.35\times 10^{-24}n\ \mathrm{g/cm^3} \\ \frac{e^2}{\hbar v_F} &= 6.1\times 10^7 n^{-1/3} = 0.915\rho^{-1/3} \end{aligned} \tag{15}$$

In obtaining these expressions we have taken $M = 2ZM_p$, where M_p is the mass of a proton. In the case of metallic hydrogen, Eq. (14) takes the form

$$T_c \approx 5\times 10^3\rho^{1/6}\exp(-3.5\rho^{1/3})$$

The factor preceding the exponential in Eq. (14) is determined only to order of magnitude, even in terms of the "jelly" model. But the factor $8/\pi$ in the exponential in Eq. (14) is determined exactly for the "jelly" model.[6]

As for the relation of form

$$kT_c \sim \hbar\omega_{0i}\, e^{-\hbar v_F/e^2}$$

this is of itself very general in character.[7] Nor can there hardly be any doubt that at high densities the critical temperature T_c will approach zero exponentially like $e^{-\alpha\rho^{1/3}}$, where α is a suitable constant.

Thus superconductivity of dense cosmic plasma, if it does occur, will essentially be found only in the range of densities that are not too high. According to Eq. (14), for $\rho = 1$, 10, 100, and 1000 g/cm^3 we have $T_c \approx 200$, 10, 10^{-4}, and 10^{-8} °K respectively. These figures speak quite eloquently. Even if the factor preceding the

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[6] The formula for $N(0) \cdot V$ in the "jelly" model, as given in Reference 6 and used in Reference 12, is not accurate and cannot be applied, particularly for $Z > 1$.

[7] As we have seen above [Eqs. (8) and (9) and the accompanying explanations], $g = N(0) \cdot V$, where $N(0) = mk_F/2\pi^2\hbar^2$ and V is some mean value of the matrix element $4\pi e^2/[q^2 \mid \epsilon(\omega, q) \mid]$ in the attraction region. The change in the momentum $\hbar q$ is in order of magnitude equal to $\hbar k_F = p_F = mv_F$, so that $g \approx k_F^2 \mid \epsilon \mid \hbar/e^2 mk_F \approx \hbar v_F \mid \epsilon \mid/e^2$, where $\mid \epsilon \mid$ is a dimensionless quantity. An estimate of T_c for strongly compressed material is also given in Reference 13.

exponential were increased by one or two orders (and we see no grounds for this), superconductivity still would not develop for $T_c > 1°K$ at densities $\rho \gtrsim 10^2$ g/cm³. A change in the exponential factor itself would of course be more important. But as already mentioned there is no special reason for such a change for $\rho \gg 1$, and above all one would evidently be concerned only with refining the coefficient of $\rho^{1/3}$. As a result for $\rho \gtrsim 10^3$ we will always have $T_c < 1°K$, in all likelihood.[8]

For temperatures $T_c > 1°K$, then, superconductivity would be possible only in the density range $\rho \lesssim 30$–100 g/cm³. Equation (14) is even less reliable in this range than at higher densities. The density range $\rho \lesssim 10$ g/cm³ corresponds to ordinary metallic superconductors, and it is well established that the critical temperature here by no means depends on the density alone, but also on the details of the lattice structure, the valance of the atoms, and so on. Cosmic conditions would differ from terrestrial conditions primarily in having a different chemical composition for the material and a considerably wider range of possible densities. Metallic hydrogen may serve as a striking example here.

At "low" pressures solid hydrogen will in a certain sense remain molecular and will be a dielectric. But at pressures $P \gtrsim 10^6$ atm the dielectric phase should be transformed into a metallic phase through a phase transition of the first kind. At low temperatures the equilibrium transition takes place[(14)] at a pressure $P_{\text{ph}} = 2.4 \times 10^6$ atm; the density of the metallic phase is $\rho_{\text{ph}} = 1.1$ g/cm³, and the temperature for electron degeneracy is $T_0 \approx 4 \times 10^3$ °K. The metastable metallic phase will possess an even lower density and at low temperatures a transformation of this metastable phase into a stable molecular phase might hardly take place at all under favorable conditions. According to a relation of the type of Eq. (14), $T_c \approx 150°K$ for metallic hydrogen; a similar estimate has been obtained by Ashcroft[(18)] (see also Ref. 43).

Whether metallic hydrogen is superconducting or not remains unknown. One would hope that before long an answer to this question might be obtained with some confidence either through calculations or by experiment. In fact, stationary pressures of about 5×10^5 atm have already been achieved in the laboratory, and it is likely that pressures as high as $(1.0$–$1.5) \times 10^6$ atm might be attainable. Moreover, pressures up to 10^7 atm and even higher have been achieved with shock waves.[(15)] Unfortunately, in work performed both with stationary equipment at ultrahigh pressure and particularly by the shock-wave method it is very difficult to detect superconductivity if the critical temperature is not too high. The progress in this area is very rapid,[(16)] however, and the possibilities for investigating the conductivity of metallic hydrogen at moderate and even at low temperatures seem not to be merely visionary.

[8] It has been claimed[(42)] that $T_c \approx \theta_D/10$ for strongly compressed material, that is, that $e^{-1/g} \approx 1/10$; consequently for $n \approx 10^{30}$ ($\rho > 10^6$ g/cm³), $T_c \approx 10^6$ °K. This conclusion seems erroneous to us. Apart from referring the reader to a number of papers[(6,7,13)] where the use of an equation of the type (14) is substantiated, we may mention the following. It is the very essence of the problem that as the electron charge e approaches zero, that is, if interaction disappears, then superconductivity should also disappear. But the result given in Reference 42 does not satisfy this obvious requirement; quite apart from the dependence of the Debye temperature θ_D on e, it is clear that $g = N(0) \cdot V \to 0$ as $e \to 0$, since g is a measure of the interaction between electrons.

The great planets of the solar system, Jupiter and Saturn, have masses of 1.9×10^{30} g and 0.57×10^{30} g respectively, or 318 and 95 times the mass of the earth. The mean densities of these planets are 1.38 and 0.71 g/cm^3, whereas the mean density of the earth is 5.5 g/cm^3. At the centers of Jupiter and Saturn[17] the density reaches 30 and 15 g/cm^3 respectively. Ashcroft[18] quotes a value of 5 g/cm^3 for the density in the central portion of Jupiter, with a temperature $T \approx 100$–200°K (see, however, Hubbard[19]). As for the chemical composition, hydrogen[17] comprises 78 % of the mass of Jupiter and 63 % of the mass of Saturn. We cannot be certain just how accurate all these values are, but in the present case it is of no particular importance. The point is that metallic hydrogen (with a considerable admixture of other elements) ought to occur in the interiors of Jupiter and Saturn with densities in the range from $\rho \approx 1$ to $\rho \approx 5$–30 g/cm^3 and not too high a temperature. Thus for planets of this type (and there can be little doubt that they would also exist in certain other planetary systems) the question of superconductivity is an entirely realistic one even though it may be answered negatively. In particular, even if metallic hydrogen were superconducting, the temperature in Jupiter and Saturn might prove to be higher than the critical temperature T_c at which superconductivity would disappear.

The temperature $T_0 \approx E_F/\hbar k$ for degeneracy of a Fermi gas is related to the concentration $n = N/V$ and density $\rho = n\mu_e M_p$ of the gas by the expressions

$$T_0 \approx \frac{(3\pi^2)^{2/3}\,\hbar^2}{2mk}\,n^{2/3} \approx 3 \times 10^5\,\frac{\rho_0^{2/3}}{\mu_e^{2/3}}\ \text{°K}$$
$$\rho_0 \approx 6 \times 10^{-9}\mu_e T_0^{3/2}\ \text{g/cm}^3 \qquad (16)$$

For hydrogen, helium, and a mixture of heavier stable elements, $\mu_e = 1$, 2, and 2.2 respectively. If $\rho > \rho_0$ and $T < T_0$ (or, strictly speaking, if $\rho \gg \rho_0$ and $T \ll T_0$) the gas will be degenerate. For example, in the sun $T_{c\odot} \approx 10^7$ °K and $\rho_{c\odot} \approx 120$ g/cm^3 at the center whereas for $T_0 = 10^7$ °K and $\mu_e = 2$ the density $\rho \approx 400$ g/cm^3 {see Greenstein,[20] where Eqs. (16) are used with a slightly different coefficient, which is of no importance}; thus for the sun one cannot speak of degeneracy of the electron gas that is at all complete. For cooler main-sequence stars the effects of degeneracy are stronger. As we have seen, however, superconductivity phenomena would not be expected for compressed material at $T \gtrsim 10^2$–10^3 °K. Thus there is no reason to expect superconductivity for any stars other than white dwarfs. White dwarfs in fact represent the final stage in stellar evolution, having a mass $M < M_{cr} \approx 1.2M_\odot$ (see above and References 11, 21, and 22), and as a result they could be cool. In most cases, to be sure, white dwarfs will have remained hot, for they will not have been able to cool off (the Galaxy and its constituent stars have existed for about 10 billion years; white dwarfs have been formed through the evolutionary process of stars in the Galaxy, so that they are younger than the Galaxy itself). Of course the white dwarfs that we actually see have not cooled off; furthermore, their surface temperature is about 10^4 °K, higher than for the sun and most other stars (it is for this reason that white dwarfs were called "white," as the light they emit is richer in blue and violet rays than is the case for yellow and red stars). But the cooling time of a white dwarf depends on its mass. As the mass M of the star approaches the critical mass M_{cr}, the density of the dwarf increases and values of $\rho_c \approx 10^9$–10^{10} g/cm³ are attained at the center of the star (the maximum density prior to the onset of instability depends on the composition of the star and certain other assumptions used in deriving the equation of state of the material; see References 20–23 and the references cited there).

For dense material the Debye temperature θ_D rises; it is given by the relation[9,20]

$$\theta_D = 1.7 \times 10^6 \frac{2Z}{A} \left(\frac{\rho}{10^6}\right)^{1/2} \tag{17}$$

For helium and heavier elements $2Z/A \approx 1$ (where A is the atomic weight) and if $\rho = 10^{10}$ g/cm³ the temperature $\theta_D \approx 2 \times 10^8$ °K, whereas for "ordinary" white dwarfs with $\rho \approx 10^6$ g/cm³ the temperature $\theta_D \approx 10^6$ °K. At temperatures T appreciably lower than θ_D the specific heat of the material is proportional to T^3 (the Debye law) and is considerably lower than the temperature-independent classical specific heat, observed for $T \gg \theta_D$. It is therefore evident that very dense white dwarfs with $T \ll \theta_D$ will have a relatively small free energy, proportional to T^4. Consequently such dwarfs will cool off more rapidly. As a result it is entirely possible that some of the densest white dwarfs will already have been able to cool to very low temperatures by the present epoch. There will of course be few such stars. It would not be possible to observe them by optical means, and naturally no type of observational evidence is available for this reason.

Might there be any way at all to observe cool white dwarfs? This question is analogous the one that is naturally posed with regard to other stars—neutron and collapsing stars—which it is impossible or very difficult to see in the optical part of the spectrum. All such stars could in principle be perceived through thair gravitational field; for example, an invisible star belonging to a binary system should influence the motion of the other star. In the case of neutron stars, if they are hot enough one might hope to detect them from their x rays as well. Finally, if white dwarfs and neutron stars oscillate (pulsate) they could be strong sources of radio emission.

The discovery of pulsars[24–27] was from the very outset related to pulsations of this kind in neutron stars or white dwarfs. For a while it seemed that the pulsating-dwarf model was preferable,[26–28] but in October 1968, after some considerably shorter oscillation periods had been discovered,[23,29,30] the situation changed. Apparently the pulsars that we observe represent oscillating and simultaneously rotating neutron stars. However, one can hardly exclude the possibility that pulsars of another type might exist and be detected, namely, white-dwarf pulsars. This is not the place to develop the topic further[27]; we merely wish to emphasize that cool white dwarfs need not by any means be considered unobservable.

As is clear from Eq. (14) and the comments already made upon it, even if superconductivity is possible in the central regions of white dwarfs the critical temperature would have to be entirely negligible. Superconductivity at $T_c \lesssim 10^2$ °K, and perhaps also at $T_c \lesssim 10^3$ °K, could in principle be observable in a suitable peripheral envelope about a cool white dwarf, somewhere in the density range $\rho \approx 1$–100 g/cm³. The chemical composition of different white dwarfs might not be the same. A similar diversity in composition might also apply to denser white dwarfs. Thus in addition to the density ρ there is another parameter, the chemical composition, and still other parameters might be associated with the type of crystal lattice.

The presence of superconductivity in one layer of a star would affect the specific heat of the layer, its heat conductivity, compressibility, and so on. From the standpoint of their influence on the parameters of the star as a whole, these variations are minor, but indirectly they could be manifested in various effects, in particular the pulsations of the star. This problem has not yet been investigated. We might incidentally mention one other very strong effect: even a thin superconducting layer will materially alter the magnetic field of a star.[12] On the other hand, the radio emission of pulsars probably is governed to a large extent by the configuration and strength of the magnetic field in the stellar atmosphere.[27,28] As a result the presence of superconductivity might significantly, or even radically, influence the radio emission of a cool, pulsating, magnetic white dwarf. In the case of Jupiter, the magnetic effect of superconductivity might also play some role,[18] since this planet apparently possesses a magnetic moment.

In considering the superconductivity of certain regions in white dwarfs and planets such as Jupiter and Saturn, however, the most important point now is not a discussion of possible consequences of the occurrence of superconductivity, but an understanding of the fundamentals—a determination of the critical temperature for metallic hydrogen and the other strongly compressed substances ($\rho \approx$ 1–100 g/cm^3) that are encountered in stars and planets.

5. SUPERFLUIDITY AND SUPERCONDUCTIVITY IN NEUTRON STARS

As early as the 1930s, soon after the discovery of the neutron and the development of the proton–neutron model for the atomic nucleus, the possible formation and existence of neutron stars came under consideration (pioneering work in this area is cited on page 225 of Reference 11; for further discussion of the properties of neutron stars see References 31 and 32). It would have been rather more accurate to speak of the neutron core of a star, for matter will consist primarily of neutrons only at densities $\rho \gtrsim 10^{12}$ g/cm^3. For this reason a neutron core will always be surrounded by a kind of "plasma" envelope, a quasineutral mixture of electrons, protons, and nuclei. The plasma envelope, particularly for pulsating stars and in view of the influence of the magnetic field (for neutron stars it could be colossal; see References 33 and 34), may far exceed the neutron core in size. At any rate this remark applys to the rarefied plasma envelope (corona). But the denser plasma envelope may also be quite large, especially for neutron stars of low mass.[23] Such a star would nevertheless be quite different from other stars, so that the term "neutron star" is reasonably accurate and definite.

Many research papers and surveys have been devoted to the theory of neutron stars.[11,31,32] Prospects for detecting such stars have been mentioned above. Indeed, at this very time, now that it has apparently become possible to identify pulsars with oscillating neutron stars, a new chapter in their study has opened up; from hypothetical objects neutron stars have now become observable celestial bodies. In this connection it is particularly important to note that neutron stars should, at least within a portion of their volume, be superfluid, and possible even superconducting.

The conclusion that neutron stars should be superfluid can be drawn from very simple considerations[35] (the possible superfluidity of the neutron core of a star was in fact first pointed out some years before[36]). It is known from neutron-scattering experiments that two neutrons with antiparallel spins will attract each other, at any rate at those distances (for those momenta) which correspond to the Fermi boundary in neutron stars ($p_F = \hbar k_F$, $k_F \lesssim 10^{13}\ \text{cm}^{-1}$). Although this attraction would not result in the formation of a stable bineutron, in the case of a degenerate neutron gas it should, in accordance with the BCS theory,[5,6] lead to the "coupling" of neutrons and to the formation of a gap in the energy spectrum of the system. If a gap is present in the spectrum the neutron fluid should be superfluid, but its superconductivity is not in question since neutrons are not charged.

To estimate the width $\Delta(0)$ of the gap and the corresponding critical temperature $T_c \approx \Delta(0)/k$, we shall use the BCS equation (9). The effective width of the energy range within which neutrons will be attracted is equal in order of magnitude to $E_0 = k\theta \approx p_F\hbar/M_n a$ where $a \approx 10^{-13}$ cm is the range of nuclear forces, $M_n \approx M_p$ is the mass of a neutron, and $\hbar/a$ is the characteristic value of the momentum transmitted in the collision of neutrons. For densities $\rho \approx 10^{13}$–10^{15} g/cm^3 the energy $E_0 \approx 5$–20 MeV. The density of states at the Fermi surface is

$$N(0) = M_n k_F/2\pi^2\hbar^2 \approx (0.5\text{–}2) \times 10^{42}$$

The most difficult quantity to estimate reliably is the matrix element of the interaction energy V appearing in the BCS formula (9). If the interaction of neutrons in the singlet state is described by a potential well of depth 15 MeV and width 2.5×10^{-13} cm, then $V = \int V(\mathbf{r})\, d^3r \approx 2 \times 10^{-42}$. It follows that

$$\begin{aligned} \Delta(0) &\approx E_0\, e^{-1/N(0)\cdot V} \approx 1\text{–}20\ \text{MeV} \\ \rho &\approx 10^{13}\text{–}10^{15}\ \text{g/cm}^3 \end{aligned} \tag{18}$$

The nuclear density $\rho_n \approx (2\text{–}3) \times 10^{14}$ g/cm^3, and in this case $\Delta \approx 5$ MeV, which does not conflict with the established findings of nuclear physics. Such a check is very important in view of the roughness of the estimates and the exponential character of Eq. (18).

A gap $\Delta \approx 1$–20 MeV corresponds to a critical temperature $T_c \approx 10^{10}$–10^{11} °K, which is considerably higher than the "ordinary" temperature of neutron stars. We thereby reach the conclusion that these stars are superfluid.

It is entirely proper, however, to inquire as to the reliability of this conclusion. As the density of the neutron fluid drops an increasing number of protons and nuclei will appear in it, so that for $\rho < 10^{12}$ g/cm^3 it would be hard to consider our estimate as even qualitatively valid. On the other hand, at high densities the interaction between neutrons cannot be regarded merely as an interaction between pairs of particles (two-particle forces), nor may collisions between neutrons be treated as pair collisions.[37,38] As a result there is reason to believe that for $\rho \gtrsim \rho_n$ the gap in the nucleon spectrum will disappear. According to Wolf[38] the gap in the neutron spectrum will close at a density $\rho = 2 \times 10^{14}$ g/cm^3, which is considered equal to $\frac{1}{2}\rho_n$. It is difficult for us to judge how accurate this estimate is. Leaving aside the quantitative aspect of the matter, we may evidently consider that in the density range from about 10^{12} to (1–3) $\times$ 10^{14} g/cm^3 a gap will be present in the spectrum, with $\Delta(0) \approx$ 1–10 MeV.[9] As applied to neutron stars, this means that they will contain a superfluid layer for $T < T_c$. For neutron stars of low mass, with $0.1M_\odot < M < (0.3\text{–}0.4)M_\odot$, such a layer may reach the center of the star, thus actually forming a sphere. For more massive neutron stars, with $(0.3\text{–}0.4)M_\odot < M < 2M_\odot$, the appearance of a non-superfluid core in the central portion will be possible, and in fact for $M \gtrsim M_\odot$ highly probable.

Other particles will also be present in a neutron fluid. If we are not concerned with the range of exceptionally high densities ($\rho > 3 \times 10^{14}$ g/cm^3), protons and electrons will be involved.[11,32] At a density $\rho \approx \rho_n \approx 2 \times 10^{14}$ g/cm^3, the number of protons will comprise roughly 1% of the number of neutrons. The number of electrons will be practically equal to the number of protons (this is clear from the requirement that the system be quasineutral, but with the assumption that a significant number of nuclei different from protons is absent). For densities $\rho \lesssim \rho_n$ that are not too high, neutrons, protons, and electrons in the normal state will, in a sense, form three coexisting degenerate Fermi gases (it would be more accurate to speak of Fermi fluids; but in this case the distinction is unimportant since the excitation spectrum of a Fermi fluid, or liquid, qualitatively coincides with the spectrum of a Fermi gas). The question now arises naturally whether the superfluidity extends not only to the neutrons but also to the protons and electrons. Properly speaking, in the case of protons and electrons one might observe superconductivity rather than supefluidity. For electrons in neutron stars the density is so high that the gap may be considered absent (see Section 4).[10] As for the protons, at a stellar density $\rho \approx \rho_n \approx 10^{14}$ g/cm^3 the density of the proton "phase"would be $\rho_p \approx 10^{12}$ g/cm^3, and superconductivity would not be excluded. Unfortunately, we are not aware of any persuasive estimates on this point (according to Wolf,[38] who emphasizes the roughness of the estimate, for $\rho = 0.75\rho_n$ the gap is less than 1 MeV, while for $\rho > \rho_n$ the gap is less than 0.2 MeV.)

What role do superfluidity and superconductivity play in neutron stars?

The presence of a gap in the spectrum alters the temperature dependence of the internal energy, and thereby also such derivative quantities as the specific heat. While the specific heat of a Fermi gas is proportional to the temperature ($c_n = \gamma T$), for the superfluid (superconducting) state the specific heat depends exponentially on T ($c_s \sim e^{-\Delta/kT}$). In analyses of the process of cooling or heating (such as through accretion of matter) neutron stars, the change in the temperature dependence of the specific heat plays an important part. It is perfectly clear that if a gap is present,

[9] It is worth emphasizing that a rigorous proof of the existence of a gap in the neutron spectrum is still lacking. The quantitative estimate of Δ will of course also need to be improved.

[10] This remark, of course, does not apply to the plasma layer near the surface of the star, where for $\rho \approx$ 1–100 g/cm^3 and $T \lesssim 10^2\text{–}10^3$ °K superconductivity could in principle be observable.

cooling will occur considerably faster, for a given heat dissipation, than for a spectrum without a gap.

A second important effect associated with the presence of a gap concerns the resultant change in the rate of the reactions cooling the star, leading to the formation of neutrinos and antineutrinos by the reactions

$$n + n \rightarrow n + p + e + \bar{\nu} \quad \text{and} \quad n + p + e \rightarrow n + n + \nu$$

(see References 37–39). Furthermore, the occurrence of a gap in the energy spectrum will affect many other quantities and processes, in particular the compressibility, the viscosity, and so on. In this connection, for analyses of the dynamical processes in a star, in particular its pulsations, the superfluidity of the star may be very important. (We do not refer here to superfluidity effects themselves, because for a massive body the critical velocity for superfluid motion should be entirely negligible(35); but an indirect influence from the presence of a large number of the vortex lines that occur in a rotating superfluid is entirely possible.) Finally, it is worth emphasizing that the superconductivity of a neutron star, or in fact even of a thin layer, may substantially affect the magnetic field of the star—its configuration, evolution, and so on. Since neutron stars are probably magnetic (they have an appreciable and sometimes a large magnetic moment; see References 33 and 34 as well as References 27–30), this last comment would seem to merit detailed investigation, as would, of course, many of the other topics discussed above.

6. SUPERFLUIDITY IN A COSMOLOGICAL NEUTRINO "SEA"

Neutrinos are the most penetrating of all known particles, or at any rate of particles having half-integral spin (we make this stipulation so as to avoid the problem of gravitons). It is in fact impossible to confine neutrinos not only under laboratory conditions but even in stellar interiors (except for collapsing stars or the hypothetical structures related to them—geons). As a result it might seem that the possibility of superfluidity for an aggregate of neutrinos would not be worth any attention or discussion. Such a conclusion, however, would not be entirely warranted: the problem of superfluidity in a cosmological neutrino "sea" is very definitely of interest.(40)

In both isotropic and anisotropic cosmological models the role of neutrinos is important in certain early evolutionary stages, and in some versions their contribution is particularly strong.(11,41) In particular, one cannot exclude the possibility that at the very time when some of the chemical elements comprising the universe were formed, neutrinos (a neutrino "sea") constituted a degenerate Fermi gas. The density corresponding to a gas of degenerate neutrinos (or antineutrinos) is given by

$$\rho_\nu = E_F{}^4/8\pi^2 c^5 \hbar \approx 3 \times 10^3\,(E_F[\mathrm{MeV}])^4 \quad \mathrm{g/cm^3} \tag{19}$$

Here E_F is the energy at the Fermi limit and E_F [MeV] is the same energy in MeV.

On the other hand, in isotropic Friedmann models (that is, in isotropic models without the Λ term) the total density in the early evolutionary stages is

$$\rho = \epsilon/c^2 = 3/32\pi G t^2 \approx (4.5 \times 10^5)/t^2 \quad \mathrm{g/cm^3} \tag{20}$$

where $G = 6.67 \times 10^{-8}$ is the gravitational constant and t is the time (in seconds) from the "beginning" of the evolution. Evidently $\rho_\nu \lesssim \rho$, so that

$$\rho_{\nu,\max} = E_{F,\max}^4/8\pi^2c^5\hbar^2 \approx 3/32\pi G t^2, \qquad E_{F,\max} \approx 3/\sqrt{t[\text{sec}]} \ \text{MeV} \tag{21}$$

As is customary, the neutrino gas has above been considered an ideal gas. However, if this gas is degenerate it might develop into a superfluid state similar to the superconducting state of electrons in a metal. A sufficient and probably a necessary condition for the appearance of such superfluidity is the presence of attraction between neutrinos. There can be no doubt that neutrinos[11] do interact somehow with each other, but neither the sign nor the order of magnitude of this interaction is yet known. One cannot exclude the existence of a "direct" interaction between neutrinos as described by a term of the form $\frac{1}{2}\lambda[\psi_\nu^*(\psi_\nu^*\psi_\nu)\psi_\nu]$ in the expression for the Hamiltonian operator. But if such a "direct" interaction is absent, an analogous term will probably appear in the Hamiltonian in higher approximations of the perturbation theory because of the known interaction of neutrinos with electrons or muons. In such an event, however, we would be dealing with a divergent expression and it would be quite impossible to determine the sign and absolute value of the coefficient λ with any confidence. Moreover, even apart from the origin of a term of the type $\lambda\psi^4$, a rigorous solution to the problem of a neutrino field with allowance for this term has not yet been obtained. In this connection the ordinary BCS theory has been applied[(40)] in a formulation where the interaction between particles is described by the Hamiltonian $\frac{1}{2}\lambda\int[\psi^*(\psi^*\psi)\psi]\,d^3r$. But while for electrons in metals the energy of electron "excitation" near the Fermi limit has the form $\xi = v_F(p - p_F)$, for neutrinos $\xi = c(p - p_F)$, where c is the velocity of light, $p_F = \hbar k_F$ is the momentum, and $v_F = p_F/m$ is the velocity at the Fermi surface. Moreover, one should recognize that for a given momentum the spin of a neutrino can have only one direction.

To find the gap in the energy spectrum of the system, in addition to assuming the presence of an attraction between particles near the Fermi energy E_F (this implies that in the expression given above $\lambda < 0$), one should also make definite assumptions regarding the width of the energy range within which the attraction occurs. In the BCS theory for metals it is considered that the region for attraction is $k\theta = \hbar\omega_c \ll E_F$. One then obtains Eq. (9), which we shall here write in the form

$$\Delta(0) \approx \hbar\omega_c \exp\left(-\pi^2 v_F^3\hbar^3/2\,|\lambda|\,E_F^2\right) \tag{22}$$

For a neutrino gas we do not know what the attraction region is, if it exists at all. But if the BCS theory is applicable in any sense, if only for orientation, then $\hbar\omega_c \lesssim E_F$. Assuming that for a neutrino "sea" $\lambda < 0$, $\hbar\omega_c \approx E_F$, and $\Delta \ll E_F$, we obtain

$$\Delta(0) \approx E_F \exp(-4\pi^2c^3\hbar^3/|\lambda|\,E_F^2) \tag{23}$$

Apart from differences in the numerical coefficient, this equation is entirely analogous to Eq. (22) with the natural replacement of v_F by c, and of course another value of $|\lambda|$.

[11] For simplicity we refer only to neutrinos, but all our remarks apply also to antineutrinos. Electron and muon neutrinos could also be considered separately, but there is no need to do so in view of the character of the discussion in this section.

For metallic superconductors with $\Delta/\hbar\omega_c \approx \Delta/k\theta_D \approx 10^{-2}$ and $v_F \approx 10^8$ cm/sec, the coefficient $|\lambda| \approx 5 \times 10^{-35}$ erg · cm³. For the universal weak interaction, $|\lambda| \approx 10^{-49}$ erg · cm³. If we adopt the same value in Eq. (23), then

$$\ln \Delta/E_F \approx -10/E_F^2;$$

for example, $\ln \Delta/E_F \approx -10$ if $E_F \approx 1$ erg $\approx 10^6$ MeV, or $\rho \approx 3 \times 10^{27}$ g/cm³, $t \approx 10^{-11}$ sec [see Eqs. (19) and (20)]. The problem of these early evolutionary stages of the universe, if they actually did occur (this depends on the range of applicability and character of the cosmological models), is of exceptional interest and at the same time remains wholly unclear. For "hot" cosmological models[11] the neutrino degeneracy would be incomplete, or even almost entirely absent. Moreover, for $\rho > 10^{14}$–10^{15} g/cm³ the equation of state of matter is known only very vaguely. On the other hand, the roughness of the calculation and the exponential character of the relation (23), together with the uncertainty of the values of $|\lambda|$ and $\hbar\omega_c$, prevents us from completely excluding the possibility that the gap in the spectrum might be considerable even for $E_F \approx 1$ MeV ($\rho \approx 3 \times 10^3$ g/cm³, $t \approx 10$ sec). The latter values represent a phase at which, or near which, certain nuclear reactions might have taken place and have determined the chemical composition of matter in the universe. The presence of a gap in the neutrino spectrum would have affected the course of these nuclear reactions. Neutrino superfluidity could in principle also be of interest for the hypothetical dense "superstars," in analyses of their collapse or "anticollapse."[11]

Without question any discussion of the problem of superfluidity in the neutrino "sea" is somewhat speculative in character. But the problem is a curious one from a purely theoretical standpoint, and indeed should not be ignored in considering various cosmological models.

7. CONCLUDING REMARKS

We have discussed or mentioned above a variety of possibilities for the occurrence of superfluidity and superconductivity under cosmic conditions. All these possibilities are of theoretical interest, but two of them are particularly important and relevant in a specifically astrophysical context.

We refer, in the first place, to the superconductivity of strongly compressed material ($\rho \approx 1$–100 g/cm³), in particular, metallic hydrogen. Material of this kind is present in the interiors of Jupiter and Saturn (and of course other similar planets), in certain peripheral layers of white-dwarf stars, and of neutron stars that are not too hot. At the current stage of development of the theory we can only make a rough estimate of the critical temperature T_c, assuming, however, that superconductivity does prevail. The main task is therefore to refine the calculations in an effort to explain the conditions under which superconductivity would appear and to obtain a more reliable estimate for T_c. One cannot exclude the possibility of an experimental approach in this direction by obtaining metallic hydrogen and other substances in the laboratory with a "cosmic" composition, under pressures greater than 10^6 atm.

Is there any hope of securing information on the superconductivity of strongly compressed matter on the basis of astronomical observations? Such a possibility seems highly unlikely, since the strongly compressed matter is located in the interiors of planets or stars. The influence of the superconductivity of such material upon the magnetic field of a planet or a white dwarf might prove to be significant, but this effect would be difficult to observe and would not be very specific in character. In other words, in the present instance as in most others, physicists might be of assistance to astrophysicists in deciphering complex phenomena, but the reverse would not be true.

The second of the important examples mentioned above is the superfluidity of neutron stars, as well as the question of the superconductivity of protons in neutron stars. There would seem to be little doubt that superfluidity does prevail in the neutron fluid in neutron stars, at least in some rather thick layer of such a star. How thick is the superfluid layer, and what are its parameters? Answers to these questions should come from nuclear physics and from calculations of the equation of state for superfluid nuclear matter. The same remark applies to an analysis of the problem of the superconductivity of protons in neutron stars. The domain of astrophysics, on the other hand, would include calculations of the cooling rate of neutron stars with allowance for their superfluidity and possible superconductivity, as well as various caculations concerning the oscillations (pulsations) of neutron stars and the variations in their magnetic field. Now that neutron stars apparently have at last been detected, after more than 30 years of discussion and search, everything associated with such stars takes on special interest. The problem of superfluidity and superconductivity in neutron stars and related questions will undoubtedly take their place at the forefront of research on these remarkable celestial bodies.

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